AN

APPENDIX

In Anſwer to the *Reaſons for not replying to Mr.* WALTON's Full Anſwer.

BY a Paper, intitled, *Reaſons for not replying to Mr.* WALTON's *full Anſwer*, I find that the Author of the *Minute Philoſopher*, ſtill perſiſts in his Miſtakes concerning Sir *Iſaac Newton*'s Doctrine of Fluxions. He declares *he can honeſtly ſay, that the more I explain, the more he is puzzled:* And I can as honeſtly ſay, that I believe him. For a Perſon who does not underſtand what Sir

Iſaac means by *Velocity*, muſt neceſſarily be ignorant of his Doctrine of Fluxions, deduc'd from the Nature of *Velocity*; and be puzzled with all true Explications of that Doctrine. What Sir *Iſaac Newton* means by the Word *Velocity*, I ſhall explain in the following Propoſitions.

I. The Velocity of a Body, is the Ratio of the Quantity of Motion to the Quantity of Matter in the Body; and is as the Ratio of the Quantity of the Action of the moving Force to the Weight of the Body; or as the Ratio of the Quantity of that Action to the Denſity and Magnitude of the Body taken together. That is, putting V for the Velocity of a Body, M for the Quantity of its Motion, F for the Quantity of the Action of the Force which generates that Motion, D for the Denſity of the Body, B for its Bulk or Magnitude and W for its Weight; V is $\frac{M}{Q}$, and is as $\frac{F}{W}$, or as $\frac{F}{DB}$.

For the Quantity of Motion is the Quantity of Matter and Velocity taken together; that is, M is QV: And conſequently V is $\frac{M}{Q}$: But the Motion of a Body is as the Quantity of the Action of the Force which generates it, Effects being always proportional to their adequate Cauſes, that is, M is as F; and at a given Diſtance from the Centre of the Earth, the Quantity of Matter in a Body, is as its Weight, or as its Denſity and Magnitude taken together; that is, Q is as W, or as DB: And therefore V is as $\frac{F}{W}$, or as $\frac{F}{DB}$.

The Forces which generate Motion in Bodies may be of different Kinds, as a Blow, Preſſure, Weight; all which may be conceived: And Denſity and Bulk may likewiſe be conceiv'd: Conſequently the two Meaſures of Velocity, $\frac{F}{W}$ and $\frac{F}{DB}$, may both be conceived, and yet neither of them includes Time and Space. This Author therefore has been groſly miſtaken in aſſerting that *Velocity* neceſſarily *implies both Time and Space, and* cannot *be conceiv'd without them.—And that there is NO Meaſure of Velocity except Time and Space, the Proportion of Velocities being ONLY compounded of the direct Proportion of the Spaces and the reciprocal Proportion of the Times.*

II. The Velocity of a Body, exiſts in the Body it ſelf while it continues in Motion.

For the Velocity of a Body, is an Effect of ſome Cauſe acting upon it; But this Effect can exiſt no where except in the Body acted upon; for could it exiſt any where elſe, an Effect might exiſt where there is no Cauſe to produce it, or in other Words, an Effect might exiſt without a Cauſe, which is abſurd: And therefore the Velocity of a Body exiſts in the Body it ſelf while it continues in Motion.

Hence it appears that a Body in Motion will have a Velocity inherent in it ſelf during the whole Time of its Motion: And conſequently there muſt be a Velocity where-ever the Body is, excluſive of Time and Space. If inſtead of a Body the Thing moved be a Point, its Velocity will exiſt in a Point, and ſucceſſively will exiſt in every Point of Space through which the Point moves.

Here I may properly take notice of this Author's Objection againſt my Proof that Velocity can exiſt in a Point; my Argument for it was conceived in theſe Words. "If a Cauſe acts continually up-
"on a given Body or Thing, in order
"to move it without any Interruption,
"there muſt enſue a continual Increaſe
"of

" of its Velocity; and conſequently no " two Points of the Space deſcribed, " however near to each other, can be " aſſign'd, in which the Velocity is the " ſame: For that wou'd manifeſtly ſup- " poſe a Ceſſation of the Action of the " moving Cauſe during the Paſſage of the " Body or Thing thro' the Interval com- " prehended between the two Points." Now he thinks that from the continual Action of a moving Force, and from the generated Velocity not being the ſame in any two different Points of the deſcribed Space, it will not follow that Velocity can exiſt in a Point of Space. But in this he is miſtaken. For the continual Action of the moving Force neceſſarily preſerves a *continual* Velocity; and if the generated Velocity be not the ſame in any two different Points of the deſcribed Space, a Velocity muſt of Conſequence exiſt in every Point of that Space.

III. The Velocity of a Body is the Rate or Degree of its Tendency forward.

For the Velocity of a Body, is a Part of its Motion by the firſt Propoſition, and exiſts in the Body, by the ſecond: But there is nothing exiſting in a Body moved or tranſlated from one Place to another beſides its Quantity of Matter, and the Rate or Degree of its Tendency forward: And therefore the Velocity of a Body is the Rate or Degree of its Tendency forward.

IV. The Account of Velocity given in the two preceding Propoſitions, is agreeable to Sir *Iſaac Newton*'s Notion of Velocity; who conſtantly excludes deſcribed Space from his Idea of that Term.

For, as I remember, whenever he uſes the Word *Velocity* in his Principles of Philoſophy, he ſpeaks of the Velocity of a Body exiſting in ſome one certain Place; by which it appears, that he confines Velocity to the Place of the Body: But there is no Space deſcribed by a moving

Body exiſting in one and the ſame Place: And therefore he excludes all Space deſcribed from his *Idea* of *Velocity*. And he expreſly does it in his Doctrine of Fluxions. For he calls Velocities *Fluxions*; and by Fluxions declares, that he does not underſtand any Increments generated, and conſequently not any Spaces deſcribed: Whence it neceſſarily follows, that Velocities, in his Doctrine of Fluxions, do not imply any Spaces deſcribed. Therefore the Propoſition is true.

The Velocity of a Body, is an Effect communicated to the Body by the Action of ſome moving Force or Cauſe, and is retain'd in it by the Inertneſs of Matter, till deſtroy'd by the Action of ſome contrary Force or Reſiſtance; and by being retain'd in it, becomes a Cauſe of the Body's going forward, or of its being tranſlated continually into a new Place: So that the continual Tranſlation of a Body into a new Place, is really an *Effect* of Velocity, which Velocity may be conceived to exiſt in the Body prior to the Progreſs of that Body, as all Cauſes may be conceiv'd to exiſt prior to the Effects produc'd by them.

V. If the Velocity of a Body or Thing moved be given, or increaſe with the Time of the Motion, it will be meaſur'd by the Space deſcribed apply'd to the Time of its Deſcription: And if it increaſes any how either regularly or irregularly, it will be meaſur'd by the firſt Ratio of the Space to be deſcribed in a given Particle of Time. If S denotes the Space deſcribed in the Time T, and $\dot{S}$ the firſt Ratio of the Space to be deſcribed in a given Particle of Time, then V will be as $\frac{S}{T}$ when the Velocity is either given, or increaſes with the Time of the Motion; and as $\dot{S}$ when it increaſes any how regularly or irregularly.

Caſe 1. If the Velocity be given, that is, if the Body conſtantly goes on at the ſame Rate, the Space deſcribed, reckoning from the Beginning of the Motion, will be as the Time of its Deſcription; and conſequently the Ratio of the Space deſcribed to the Time of its Deſcription; that is, $\frac{S}{T}$ will be a given Quantity: But one given Quantity may be the Meaſure of another: And therefore V will be meaſur'd by $\frac{S}{T}$.

Caſe 2. If the Velocity increaſes with the Time of the Motion, the Space deſcrib'd muſt increaſe with the Square of the Time; whence $\frac{S}{T}$ will be as T, that is, as V: And conſequently in this Caſe alſo V will be meaſur'd by $\frac{S}{T}$.

Caſe 3. If the Velocity increaſes any how either regularly or irregularly, the Space to be deſcribed in a given Particle of Time, will begin to exiſt, no Part of it being yet deſcribed, with the Ratio of the Velocity: But the Ratio with which that Space begins to exiſt or to be deſcribed, is its firſt *Ratio:* And therefore the Velocity will be meaſured by the firſt Ratio of the Space to be deſcribed in a given Particle of Time. Conſequently V in all Caſes will be meaſur'd by $\dot{S}$.

The two firſt Caſes of this Propoſition are particular ones, and obtain *only* when the Force which generates the Motion acts either by one ſingle Impulſe, or continually with the ſame Degree of Strength during the whole Time of Motion: The laſt is a general one, and obtains in all Caſes whatever: This general Meaſure of Velocity Sir *Iſaac Newton* uſes in his Doctrine of Fluxions: It continually exiſts in the final Limits or Extremities of Quantities actually generated by Motion, and juſt beginning to be increas'd by a Continuance of that Motion, tho' as yet no Parts of their iſochronal Increments are deſcribed; for thoſe Increments *muſt* begin to exiſt in the Ratio of the Velocities, which the generating Quantities have in the final Limits of the Quantities generated, which are the initial Limits of their Increments. If the Ratio of the Velocities exiſting in any two Points be $\frac{4}{3}$ or $\frac{3}{2}$, the firſt Ratio of iſochronal Increments commencing from thoſe Points, will be $\frac{4}{3}$ or $\frac{3}{2}$.

Having ſhewn what Sir *Iſaac Newton* means by the Word *Velocity*, and given an Account of its *Meaſures*; I will now, by way of Inference, ſhew the Weakneſs of this Author's Objections againſt the ſeveral Parts of the foregoing Anſwer; and in doing of this I ſhall be leſs methodical, becauſe I intend to purſue *him* in the order of his *Reaſons*.

Firſt then, in *Sect.* 3. he takes notice of my Freedom in calling his *Analyſt* a Libel: I am ſorry my *free Manner* ſhould offend him; but I muſt continue to call it a Libel, till he produces Proofs to the contrary. He tells me, *I well know a bad Vindication is the bittereſt Libel.* I cannot ſay I know it; but I know that defaming one of the greateſt and beſt

Men, who had nothing at heart beſides the Promotion of true Philoſophy and true Religion; is a moſt *bitter* Libel: A bad Vindication may proceed from a good Mind and an honeſt Intention; but Defamation and Detraction can ariſe from neither.

In *Sect.* 4, 5. he finds fault with my Proof that Velocity can exiſt in a Point of Space. But I have ſhewn the Juſtneſs of that Proof under the ſecond Propoſition, to which I refer him. That Proof may perhaps appear more evident to ſome, by conſidering the Motion of a Body in falling from a State of Reſt by the Force of its own Weight. For ſetting aſide the Reſiſtance of the Air, and ſuppoſing the Weight of the Body to be the ſame at all near Diſtances from the Surface of the Earth; its Velocity, from the continued Action of that Weight, will increaſe continually as the Time of falling increaſes, or as the ſquare Root of the Length deſcribed increaſes, reckoning that Length from the Beginning of the Motion; and therefore Velocity will exiſt and be different in every different Point of the deſcribed Space: Both theſe are neceſſary and inevitable Conſequences of the continued Action of the Weight of the Body during the Time of its falling; and of the Velocity's being proportional to the ſquare Root of the Space deſcribed.

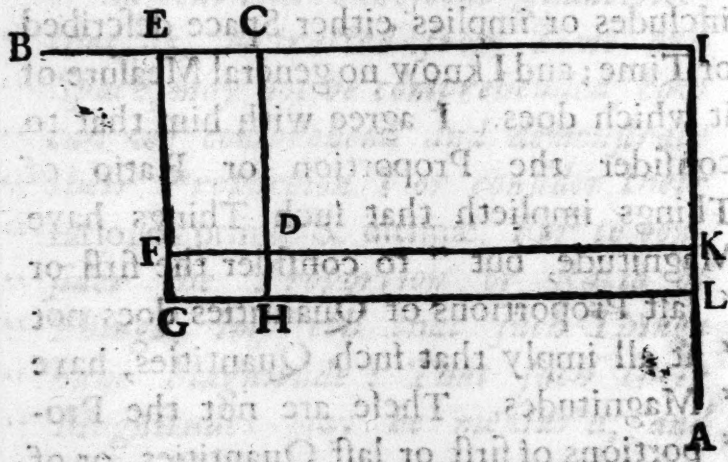
In *Sect.* 6, 7. he charges me with giving an Account of Motion different from Sir *Iſaac Newton*, "*who diſtinguiſheth two Sorts of Motion, abſolute and relative. The former he defineth to be a Tranſlation from abſolute Place to abſolute Place, the latter from one relative Place to another. Mine, which exiſts in a Point, which may be conceiv'd without Space deſcrib'd,* he ſays, *is plainly neither of theſe Sorts of Motion, but ſome third Kind, which he is at a loſs to comprehend. But he can clearly comprehend that, if we admit Motion without Space, then Sir* Iſaac Newton*'s Account of it muſt be wrong: For Place by which he defines Motion is, according to him, a Part of Space.*"

" *Space.*" Now in Anſwer to this I ſay that all Motion, which can neither be generated nor changed but by Forces impreſs'd on a Body, neceſſarily exiſts in the Body it ſelf while it continues to change its Place, and is the Quantity of Matter moved and its Velocity or Degree of Tendency forward taken together: The continual Tranſlation of a Body therefore into a new Place is, as I have before obſerved, an *Effect* of this Tendency forward in the Body, and not the Tendency itſelf; conſequently Space deſcrib'd is an *Effect* of Velocity, and not the Velocity itſelf. Velocity, according to Sir *Iſaac Newton*, does not neceſſarily imply any Space deſcribed: And therefore, conſider'd as an Effect exiſting in the Body, can be nothing but the Rate or Degree of its Tendency forward, as I have proved it to be in Prop. III. Conſequently I have not given a different Account of Motion from Sir *Iſaac Newton*, but an Account every way conſiſtent with his Principles.

In *Sect*. 8. I find the following Paſſage tranſcribed from the *Analyſt*. " *Ve-*
" *locity* neceſſarily *implies both Time*
" *and Space, and* cannot *be conceiv'd*
" *without them. And if the Velocities*
" *of naſcent and evaneſcent Quantities,*
" *that is, abſtracted from Time and*
" *Space, may not be comprehended, how*
" *can we comprehend and demonſtrate*
" *their Proportions? or conſider their*
" rationes primæ & ultimæ. *For to con-*
" *ſider the Proportion or Ratio of*
" *Things, implieth that ſuch Things*
" *have Magnitude: That ſuch their*
" *Magnitudes may be meaſur'd, and*
" *their Relations to each other known.*
" *But, as there is NO Meaſure of Ve-*
" *locity except Time and Space, the*
" *Proportion of Velocities being ONLY*
" *compounded of the direct Proportion*
" *of the Spaces and the reciprocal Pro-*
" *portion of the Times; doth it not*
" *follow, that to talk of inveſtigating,*
" *obtaining and conſidering the Pro-*
" *portions of Velocities, excluſively of*
" *Time and Space, is to talk unintel-*
" *ligibly?*" This Paſſage I have fully anſwer'd,

anſwer'd, in having proved that there are other Meaſures of Velocity beſides Time and Space. I have given two general Meaſures of it in the firſt Propoſition, and one in the fifth; all of which may be clearly conceiv'd, and yet not one of them includes or implies either Space deſcribed or Time; and I know no general Meaſure of it which does. I agree with him that to conſider the Proportion or Ratio of Things implieth that ſuch Things have Magnitude, but "to conſider the firſt or "laſt Proportions of Quantities does not "at all imply that ſuch Quantities, have "Magnitudes. Theſe are not the Pro"portions of firſt or laſt Quantities, or of "any generated Magnitudes whatever, "but the Proportions of the Velocities "with which Quantities begin or ceaſe "to have Magnitudes.

The next two Sections relate to the Moment of a Rectangle. If the Rectangle CDK be generated by the Motions of two right Lines, and from a Continuance of their Motions be increaſed by the Gnomon CGK; and if A and B are put for the Sides DK and DC, and $\frac{a}{b}$ for the firſt Ratio of their iſochronal Increments $\frac{DF}{DH}$, which firſt Ratio is equal to the Ratio of the Velocities in D to-



wards F and H; the Moment or Mutation of the Rectangle C D K, will be $A b + B a$, as I have fully proved in the foregoing Anſwer. In the Augmentation of a Rectangle no Motions exiſt but the Motions of its Sides, which Motions, according to Sir *Iſaac Newton*, conſtitute its Moment or Mutation: And if the Sides of the Rectangle E G L, flow back till they coincide with thoſe of the Rectangle C D K, and the generated Gnomon C G K vaniſhes; I have proved that the Motion ſubſiſting in the Inſtant of its Evaneſcence, which is alſo the Moment of the Rectangle C D K, is the Sum of the Motions of the Sides of that Rectangle, or A b+B a; and he ought either to allow it or prove the contrary. To ſay that a and b muſt denote the Magnitudes of the Increments D F and D H, and not their firſt Ratio, or the Ratio of the Velocities in D towards F and H, is to aſſert a manifeſt Falſhood. For the Velocities during the Paſſage of the moving Quantities from D to F and H, may be ſo changed that the Magnitudes of the Increments D F and D H, ſhall have a Proportion very different from that of the Velocities in D towards F and H: And therefore D F and D H, how ſmall ſoever, cannot meaſure the Fluxions of D K and D C; and conſequently cannot be the Things expreſſed by a and b.

In the next place he charges me with *explaining Fluxions by the Ratio of*

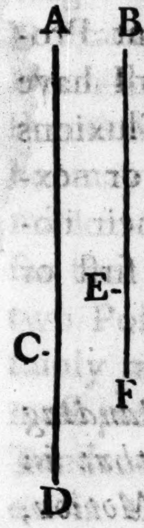
Magnitudes infinitely diminiſh'd, although I had expreſly told him, that they are not meaſured or expounded by the Proportions of any generated Magnitudes whatever; but by the firſt or laſt Proportions of iſochronal Increments generated or deſtroyed by Motion. The Paſſage which occaſion'd this ſenſeleſs Notion, is contain'd in the 9th Page of the *Vindication*, and ſtands in theſe Words. "To "obtain the laſt Ratio of ſynchronal In"crements, the Magnitudes of thoſe In"crements muſt be infinitely diminiſh'd; "For their laſt Ratio is the Ratio with "which they vaniſh and become no"thing." To which may be added a like Paſſage in the 14th Page of the ſame *Vindication*, in theſe Words. "The "Magnitudes of ſynchronal Increments "muſt be infinitely diminiſh'd and be"come evaneſcent in order to obtain "their firſt or laſt Ratios, to which Ra"tios the Ratios of their correſponding "Fluxions are equal." Now a fair and ingenuous Reader, who had nothing at heart but the eſtabliſhing of Truth and

Science, would eaſily have collected from either of theſe Paſſages, that I did not explain or expound Fluxions by the Ratio of Magnitudes infinitely diminiſh'd, but by the firſt or laſt Ratios of Increments generated or deſtroyed in equal Times; that is, by the Ratios of the Velocities with which thoſe Increments begin or ceaſe to exiſt.

In the 20th Page of the foregoing Anſwer, I have aſſerted, that "the firſt or "laſt Proportions of iſochronal Incre-"ments, ſubſiſt when the Increments "themſelves have no Magnitudes; for-"aſmuch as the Motions ſubſiſt with "which thoſe Increments, *juſt now*, *in* "*this very Inſtant*, begin or ceaſe to "exiſt; to which Motions theſe Ratios "are proportional." In Anſwer to this the Author of the *Reaſons* asks his Reader, "*Whether the iſochronal Incre-*"*ments themſelves ſubſiſt when they* "*have no Magnitude? Whether there* "*can be an Increment where there* "*is no Increaſe, or Increaſe where* "*there*

" *there is no Magnitude? Whether if* " *Magnitudes be not generated by Motion in equal Times, what elſe is generated therein, or what elſe is it* " *that Mr.* Walton *calls iſochronal?*" To the two firſt of theſe Queſtions I reply in the negative, and to the laſt in the affirmative; and would fain know what he infers from thence. I did not ſay that iſochronal Increments ſubſiſt when they have no Magnitudes, but that the firſt and laſt Ratios of ſuch Increments ſubſiſt when the Increments themſelves have no Magnitudes; and gave him the Reaſon why they ſubſiſt, which according to his uſual Candour he has taken no notice of.

If the Lines AC and BE be generated by the Motions of two Points ſetting out from A and B, and be increas'd in equal Times from a Continuance of their Motions by the Increments CD and EF; theſe Increments will begin to exiſt with the Velocities which the moving Points have in their initial Limits at C and E; and the Ratio of thoſe Velocities will be the Ratio with which the Increments begin to exiſt, that is, their firſt Ratio; conſequently the firſt Ratio of C D and E F ſubſiſts when the Increments themſelves do not ſubſiſt, that is, before they have acquired any Magnitude whatever. The firſt Ratio of the Increments C D and E F, is not the Ratio of thoſe Increments, nor of any Parts of them; it is not their Ratio conſidered as exiſting and having finite Magnitudes, but the Ratio with which they begin to exiſt and have Magnitudes. If the Velocities in C and E be as 3 and 2, the firſt Ratio of the iſochronal Increments C D and E F, will be the Ratio of 3 to 2, and may be expounded by any two Lines which are as 3 to 2; but thoſe Lines are not the Increments themſelves nor any Parts of them. For the Velocities may perpetually vary in the Progreſs of the moving Points from C and E, to D and F; and their Variation may be



be ſuch that C D and E F, however ſmall, may be in a very different Proportion from that of 3 to 2, as I have obſerved above: So that the Fluxions of Lines cannot be meaſured or expounded by any Proportions of their iſochronal Increments, except their firſt or laſt Proportions.

In order to aſſiſt the Underſtanding of his Correſpondent he ſays, that in the Geneſis of a Rectangle by Motion, I have ſuppos'd two Points to exiſt at the ſame Time in one Point, and to be moved different Ways without ſtirring from that Point. This he is pleas'd to call a *Riddle*, and ſays that *I have the Conſcience to call it a full and clear Anſwer* to Part of his CATECHISM. I do not indeed profeſs much Skill in making or explaining of *Riddles*, but I can ſee it will not be very difficult to clear this Affair. Let any two Lines KF and CH, interſecting each other in D, be deſcrib'd by the Motions of two Points, ſetting out from K and C; (*See the Figure in* p. 50.) and let the Velocities of the moving Points be ſuch, that they arrive at D exactly at the ſame Inſtant of Time: Upon their Arrival in D, it is plain the two Points will exiſt in the Point D; which he may venture to allow, ſince mathematical Points have no Magnitude: So far the *Riddle* is clear. And ſince the two Points can exiſt in D, as they certainly may without any Abſurdity, they muſt preſerve their reſpective Velocities or Tendencies forward to F and H, and go on in the Directions DF and DH, by Virtue of thoſe Velocities or Tendencies, there being nothing to change or deſtroy them: But they cannot go on in thoſe Directions without ſtirring from the Point D: And therefore he has been miſtaken in ſaying I ſuppoſed two Points to be moved different Ways in the ſame Point, without ſtirring from that Point. Farther, from the Geneſis of the Rectangle CDK, it is obvious that the common Interſection of the moving Lines IB and IA, in which the two Points are ſuppoſed to exiſt at the ſame Time, will continually go forward with the Lines themſelves; but it cannot continually go forward with the Lines themſelves unleſs it continually goes into a new Place. Surely this Author's Friends muſt be aſhamed of him for calling this a Riddle, which is plain to any one who has in the leaſt conſidered theſe Things.

In *Sect.* 14. he honeſtly confeſſes *he does not underſtand Sir* Iſaac*'s Doctrine ſo far as to frame a poſitive Idea of his Fluxions; and yet from certain negative Conceptions thereof* he takes upon him to ſay *that by the Geneſis of a Cube I have not explain'd the Nature of ſecond, third and fourth Fluxions in a Way agreeable to that Doctrine.* But in this he is miſtaken. For (by *Lem.* 2. *L.* 2. *Princip. Newt.*) if A flows with an uniform Velocity expreſs'd by a, $3aA^2$ is the Moment of A^3, $6a^2A$ the Moment of $3aA^2$, $6a^3$ the Moment of $6a^2A$, and 0 the Moment of $6a^3$: But according to Sir *Iſaac* the Moment of the Moment of A^3, is the ſecond Moment of A^3; the Moment of that ſecond Moment, is its third Moment; and the Moment of the third Moment, is its fourth Moment: And therefore $3aA^2$, $6a^2A$, $6a^3$ and 0, are the firſt, ſecond, third, and fourth Moments of A^3. Farther, I have ſhewn that all theſe Moments or Motions exiſt and may be clearly and diſtinctly conceiv'd, in the Inſtant that a Cube generated by an uniform Motion, begins to be increas'd by a Continuance of that Motion: And therefore in the Syſtem of Motion whereby a Cube begins to be augmented, I have given an Explanation of firſt, ſecond and third Fluxions every way agreeable to Sir *Iſaac*'s Doctrine: And he ought either to allow it or ſhew the contrary.

He tells his Reader (Sect. 17.) that in ſaying there can be no fourth Fluxion of a Cube, *I make it my Buſineſs directly to overthrow Sir* Iſaac Newton*'s Doctrine. For if there can be no fourth Fluxion of a Cube, there can be no ſecond Fluxion of a Line, and* a fortiori, *no third, fourth or fifth Fluxion.* Here it muſt be obſerv'd that one Circumſtance which I particularly ſuppos'd, has not been attended to; and by not attending to it, the Reader is made to believe that, according to the Account I have given of Fluxions, there can be no ſecond, third or fourth Fluxion of a Line. Now, in the 25th Page of the foregoing Anſwer, I ſuppos'd the Cube to be generated by an UNIFORM Motion; in which Caſe the Velocity expreſs'd by a, will be a given Quantity: But there can be *no kind* of Mutation or Change of a Velocity which is given; and conſequently no ſecond Fluxion of a Line, nor any fourth Fluxion of a Cube, which increaſes with ſuch a Velocity. In this therefore I have not endeavoured *directly to overthrow Sir Iſaac's Doctrine*; *I have not, to the great Relief of the learned World, deſtroy'd an indefinite Rank of Fluxions of different Orders*; nor have I given an Account of them any way inconſiſtent with Sir *Iſaac*'s Doctrine, as this Author has falſely aſſerted.

He ſays, *Sect*. 18. that *I give up Sir* Iſaac'*s Doctrine of Fluxions, and inſtead thereof humorouſly ſubſtitute what all the World knew before he was born, to wit, the three Dimenſions of a Cube and the Geneſis thereof by Motion.* But this is a Miſtake. For I neither give up the Doctrine of Fluxions, nor ſubſtitute the Geneſis of a Cube by Motion inſtead of it: I did not introduce that Geneſis in order to ſet aſide the Doctrine of Fluxions; but to exhibit a Syſtem of Motion which might explain and illuſtrate firſt, ſecond and third Fluxions, ſo as to make their Exiſtence conceivable to the meaneſt Capacity; and thereby to expoſe the Weakneſs and Ignorance of this Writer: And I have ſucceeded in both; although to his *diſcerning Eye* it may ſeem to be a *diſguiſing and betraying and giving up the Doctrine*. According to that Doctrine, the Fluxions and Moments of generated Quantities, are the Velocities and Motions with which the iſochronal Increments of thoſe Quantities begin or ceaſe to exiſt; and though the Generation of a Cube by Motion, might be known *before ever Sir* Iſaac *thought of his Fluxions*; yet it was not known, before he publiſh'd the Principles of that Doctrine, that three diſtinct Orders of Fluxions or Moments, depending upon each other, exiſted in and might be clearly and diſtinctly conceiv'd, by attending to that well known and obvious Syſtem of Motion, which exiſts in the Inſtant a generated Cube begins to be augmented.

In *Sect.* 20. *He intreats me to explain whether Sir* Iſaac's *Momentum be a finite Quantity, or an Infiniteſimal, or a mere Limit.* I tell him, that Sir *Iſaac*'s *Momentum* is a FINITE QUANTITY; it is a Product contained under the moving Quantity and its Velocity, or under the moving Quantity and firſt Ratio of the Space deſcribed by it in a given Particle of Time; the Velocity being meaſur'd by the firſt Ratio of that Space. Now the Velocity of the moving Quantity, and firſt Ratio of the Space deſcribed by it in a given Particle of Time, being both of them *finite* Quantities, may both be expreſs'd by one and the ſame Line of a *finite* Magnitude; but that Line does not exiſt in the Quantity generated or augmented by Motion. The moving Quantity exiſts both in the Fluent and in the Increment of that Fluent, being the final Limit of the former and the initial Limit of the latter: But the Line which expreſſes the Velocity, or firſt Ratio of the Space deſcribed by the moving Quantity in a given Particle of Time, does not exiſt either in the Fluent or in its Increment. By Moments therefore he is not to underſtand generated Increments of Fluents, but certain FINITE PRODUCTS OR QUANTITIES of a very different Nature from generated Increments, expreſſing only the Motions with which thoſe Increments begin or ceaſe to exiſt.

To conclude, if this Author intends to give himſelf or me any farther Trouble, he muſt conſider theſe Principles a little better than he has hitherto done, if he expects a Reply. For *I ſhall ſcarce*

ſcarce think it worth my while for the future *to beſtow a ſerious Thought on any Writer, who ſhall* dare to ſay that *Sir* Iſaac Newton *had no clear and ſteady Notions of his* Moments *and* Fluxions, *and yet leave* what has been offered by *Philalethes Cantabrigienſis* and me, in Defence of that great Man, without a fair and candid Anſwer.

FINIS.

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